APPLYING THE THERMODYNAMICS OF NONEQUILIBRIUM PROCESSES TO THE STUDY OF NONSTEADY FILTRATION IN A CRACKED-POROUS STRATUM

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Nonsteady filtration with double porosity is studied on the basis of the thermodynamics of irreversible processes. We present the basic differential heat- and mass-transfer equations.

The production of petroleum and natural gas from cracked-porous deposit sites is presently of great importance to the national economy. As is well known, such sites are found in the form of two porous media, with pores of various sizes. One is a porous medium in the conventional sense of the word ("porous block"), while the other can be classified as a porous medium in which cracks function in the role of the pore channels, with the role of the grains played by the block of rock. In this case we are thus dealing with a three-component thermodynamic system whose first component is the solid phase, while the two other phases are made up of the liquid saturating the space within the porous block and the crack space. Since the pressure and temperature of the liquid within the block and within the cracks are different, transfer processes arise between the liquid within the blocks and the liquid within the cracks (a leakage of liquid from the block to the crack, heat transfer, etc.), which is characteristic for the phenomenon under consideration. Filtration processes in producer rocks exhibiting double porosity are, as we can see, complex macroscopic processes involving viscous flow, diffusion, heat transfer, and the like.

In the following, based on the thermodynamics of nonequilibrium processes, we investigate the unique features of the energy (heat) and mass transfer involved in filtration in rocks with double porosity.

A system of differential interrelated mass- and heat-transfer equations can be solved in certain special cases to the very end. We will assume in the formulation of the heat problem that the producer site (the solid component) is linearly deformable (the porosity is variable and its variation is linearly associated with the change in pressure), while the liquid in the blocks and in the cracks is compressible. For an absolutely rigid skeleton (nondeformable) we must assume that $w_1 = 0$.

The thermodynamic methods of studying filtration in a conventional porous medium are applied in [1-4], etc. From among the cited sources we must make special reference to [2] which gives the classical theory of thermodynamics for irreversible processes and in which solutions are given for the most important problems of nonsteady heat and mass transfer

According to [2, 5], the total system of hydrodynamic equations for multicomponent systems consists of the equations of continuity, the equations of motion, the equations of energy and the phenomenological equations associating the thermodynamic flows and forces, with consideration given to limitations based on the Curie theorem.

According to [2, 5], the equation for the conservation of matter in the case of the i-th component of the mixture can be presented in the form

$$\rho \frac{d\rho_{i0}}{dt} = -\operatorname{div} j_i + I_{ij}. \tag{1}$$

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The differential equation (1) is written in a center-of-center mass system and differs from the usual equation of continuity because of the presence of the source of the i-th component. For the liquid in the porous blocks we obtain from (1)

$$\frac{\partial (m_2\rho_2\theta_2)}{\partial t} + \operatorname{div}(m_2\rho_2\theta_2 w) + \operatorname{div} j_2 - I_{23} = 0, \tag{2}$$

while for the liquid saturating the crack space we have

$$\frac{\partial (m_3\rho_3\theta_3)}{\partial t} + \operatorname{div}(m_3\rho_3\theta_3 w) + \operatorname{div} j_3 + I_{32} = 0. \tag{3}$$

For the first deformable (solid) component we have

$$\frac{\partial (m_1\rho_1)}{\partial t} + \operatorname{div}(m_1\rho_1 w) + \operatorname{div} j_1 = 0. \tag{4}$$

We note that in (2) and (3) we find the terms I_{23} and I_{32} which represent the mass sources for the components (the second and third components) as a consequence of the processes occurring within the isolated volume (leakage from the blocks to the cracks). For this leakage equation we can take the following linear relationship:

$$I_{23} = I_{32} = \alpha (P_2 - P_3), \tag{5}$$

where the coefficient α is expressed in units of h/m^2.

In compiling the equations of motion for the second and third components, we should take into consideration the fictitious mass forces which replace the effect of viscosity [6]. We are dealing here with the fact that even in the simplest cases of filtration we require integration of the Navier–Stokes equations under extremely complex boundary conditions, and we therefore resort to such an artificial method when we utilize the Euler equations, with addition of the fictitious force of viscous friction [6].

The term "fictitious force" is exceedingly conditional, since from the thermodynamics viewpoint it is a flow of vector intensity which is a function, as will be demonstrated below, of $(w_i - w_1)$.

Considering the above, we derive the equations of motion:

$$\rho_2\theta_2 \frac{dw_2}{dt} = -\operatorname{grad} P_2 + I_{23}(w_2 - w_3) - \frac{R_2}{m_2} + \rho_2 F_2; \tag{6}$$

$$\rho_3\theta_3 \frac{dw_3}{dt} = -\operatorname{grad} P_3 - I_{32}(w_2 - w_3) - \frac{R_3}{m_3} + \rho_3 F_3. \tag{7}$$

The second terms in the right-hand members of (7) and (6) characterize the momentum transfer resulting from the leakage.

For the deformable solid component the equation of motion has the form

$$m_1\rho_1 \frac{dw_1}{dt} - \operatorname{div}(m_1 \Pi) - (P_2 \operatorname{grad} m_2 + P_3 \operatorname{grad} m_3) = \sum_{i=2}^{3} R_i + \rho_1 F_1. \tag{8}$$

The second term in (8) represents the divergence from the stress tensor in the elastic-deformable solid components, while the third term is associated with the variation in the porosity of the block (m_2) and the porosity of the crack (m_3). For linear elastic strains [6] we can assume that

$$dm_2 \approx \beta_2 dP_2; \quad dm_3 \approx \beta_3 dP_3.$$

In this case the body force in (8)-(6) is the force of gravity.

The equation for the balance of the total energy for the components of the thermodynamic system can be presented in the following form:

for the first (solid) component

$$\frac{\partial}{\partial t}\left[m_1\rho_1 \left(\frac{1}{2} w_1^2 + \Psi_1 + u_1\right)\right] + \operatorname{div}\left[m_1\rho_1 \left(\frac{1}{2} w_1^2 + \Psi_1 + u_1\right) w_1\right] - \operatorname{div}(m_1 \Pi w_1) + \operatorname{div} j_q^{(1)} - \delta A_1 - q_1 = 0; \tag{9}$$

for the second component (it is analogous for the third component):

$$\frac{\partial}{\partial t}\left[m_2\rho_2\theta_2\left(\frac{1}{2}w_2^2+\Psi_2+u_2\right)\right]+\operatorname{div}\left[m_2\rho_2\theta_2\left(\frac{1}{2}w_2^2+\Psi_2+u_2\right)w_2\right]$$
$$+\operatorname{div}(m_2P_2w_2)+\operatorname{div}(m_2\theta_2\Psi_2j_2)+\operatorname{div}j_q^{(2)}+\delta A_2+q_{21}+q_{23}=0. \tag{10}$$

The third terms in (9) and (10) represent the components of the total-energy flux governed by the mechanical work performed on the components of the system; the fourth term in (9) and the fifth term in (10) represent the transfer of heat resulting from the temperature difference and from diffusion.

The quantities q_{ij} characterize the intensity of the flow of heat between the components of the system if their temperatures are not identical. Finally, the fourth term in (10) is the component of the total-energy flux due to the transfer of potential energy in the case of diffusion in the gravity field.

We note that in (9) and (10) the quantity δA_i denotes the work of the forces on the inside surfaces of separation between the liquid and the solid components in units of time ($\delta A_1 = \delta A_2 + \delta A_3$), with this work vanishing if the surface of separation is nonmoving (the nondeformable solid component).

Proceeding from the entropy balance equation [2]

$$\frac{\partial\rho s}{\partial t}=-\operatorname{div}j_{s,a}+I_s \tag{11}$$

and the Hibbs equation [2], we have the following balance equation for the first component:

$$\frac{\partial}{\partial t}(m_1\rho_1s_1)=-\operatorname{div}\left(m_1\rho_1s_1w+\frac{j_q^{(1)}-\mu_1j_1}{T_1}\right)+\frac{q_1}{T_1}-\frac{1}{T_1}\Pi\,..\,\nabla w-\frac{1}{T_1^2}\cdot j_q^{(1)}\cdot\operatorname{grad}T_1-\frac{1}{T_1}\left[j_1\cdot\left(T_1\operatorname{grad}\frac{\mu_1}{T_1}-F_1\right)\right]. \tag{12}$$

For the second component (and analogously for the third component):

$$\frac{\partial}{\partial t}(m_2\rho_2\theta_2s_2)=-\operatorname{div}\left(m_2\rho_2\theta_2s_2\cdot w+\frac{j_q^{(2)}-\mu_2j_2}{T_2}\right)$$
$$-\frac{1}{T_2}P_2\cdot\nabla w-\frac{1}{T_2^2}\cdot j_q^{(2)}\cdot\operatorname{grad}T_2-\frac{1}{T_2}\left[j_2\cdot\left(T_2\operatorname{grad}\frac{\mu_2}{T_2}-F_2\right)\right]-\frac{q_2}{T_2}+\frac{R_2(w_2-w_1)}{T_2}. \tag{13}$$

For the entire thermodynamic system, on the basis of (12) and (13) we have

$$\frac{\partial}{\partial t}\left(\sum_{i=1}^{3}m_i\rho_i\theta_is_i\right)=-\operatorname{div}\left(\sum_{i=1}^{3}m_i\rho_i\theta_is_iw+\frac{j_q^{(i)}-\mu_ij_i}{T_i}\right)-\sum_{i=1}^{3}\frac{1}{T_i^2}\cdot j_q^{(i)}\cdot\operatorname{grad}T_i-\sum_{i=1}^{3}\frac{1}{T_i}\left[j_i\cdot\left(T_i\operatorname{grad}\frac{\mu_i}{T_i}-F_i\right)\right]$$
$$+\sum_{i=2}^{3}\frac{R_i(w_i-w_1)}{T_i}+\frac{q_1}{T_1}-\frac{q_2}{T_2}-\frac{q_3}{T_3}-\frac{1}{T_1}\Pi\,..\,\nabla w-\sum_{i=2}^{3}\frac{1}{T_i}P_i\cdot\nabla w. \tag{14}$$

In the notation of (14) the first term in the right-hand member represents the entropy flux, while the remaining terms correspond to the intensity of the entropy source. We note that the intensity of the entropy source is determined by the heat conduction, diffusion, and viscosity phenomena and is presented in bilinear form, with one of the factors in each term a quantity of the flow type, while the other factor corresponds to the thermodynamic force.

To close the system of balance equations and the entropy balance, we should coordinate the thermodynamic flows and forces, bearing in mind in this case the Curie principle with respect to the identity of tensor dimensionalities for flows and forces.

For the sake of simplicity we will subsequently assume that $T_1 = T_2 = T_3 = T$; then (14) assumes the following form:

$$\frac{\partial}{\partial t}\left(\sum_{i=1}^{3}m_i\rho_i\theta_is_i\right)=-\operatorname{div}\left(\sum_{i=1}^{3}m_i\rho_i\theta_is_i\cdot w+\frac{1}{T}\cdot j_q-\frac{1}{T}\mu_ij_i\right)-\frac{1}{T^2}j_q\cdot\operatorname{grad}T-\frac{1}{T}\sum_{i=1}^{3}j_i\left(T\operatorname{grad}\frac{\mu_i}{T}-F_i\right)$$
$$+\frac{R_2(w_2-w_1)}{T}+\frac{R_3(w_3-w_1)}{T}-\frac{1}{T}(\Pi+P_2I+P_3I)\,..\,\nabla w. \tag{15}$$

Here I is the unit tensor.

If we write out all of the phenomenological equations, the following relationships will exist between the phenomenological coefficients of these equations, in accordance with the Onsager theorem:

$$L_{12}=L_{21};\quad L_{13}=L_{31};\quad L_{32}=L_{23},\ \ldots$$

The phenomenological equations will be constructed in the manner of the following (for heat flow):

$$j_q = -L_{11}\frac{\operatorname{grad} T}{T^2} - L_{12}\frac{(\operatorname{grad}\mu_1)_T}{T} - L_{13}\frac{(\operatorname{grad}\mu_2)_T}{T} - L_{14}\frac{(\operatorname{grad}\mu_3)_T}{T} + L_{15}\frac{w_2 - w_1}{T} + L_{16}\frac{w_3 - w_1}{T}. \tag{16}$$

We note that if the forces are conservative, we must take into consideration

$$F_i = -\operatorname{grad}\Psi_i.$$

The equation for the diffusion flow of the first component is given by

$$j_1 = -L_{21}\frac{\operatorname{grad} T}{T^2} - L_{22}\frac{(\operatorname{grad}\mu_1)_T}{T} - L_{23}\frac{(\operatorname{grad}\mu_2)_T}{T} - L_{24}\frac{(\operatorname{grad}\mu_3)_T}{T} + L_{25}\cdot\frac{w_2 - w_1}{T} + L_{26}\cdot\frac{w_3 - w_1}{T}. \tag{17}$$

There is no difficulty in deriving the equations for the diffusion flows j_2 and j_3. For R_2 (analogously for R_3) we have

$$R_2 = -L_{51}\frac{\operatorname{grad} T}{T^2} - L_{52}\frac{(\operatorname{grad}\mu_1)_T}{T} - L_{53}\frac{(\operatorname{grad}\mu_2)_T}{T} - L_{54}\frac{(\operatorname{grad}\mu_3)_T}{T} + L_{55}\frac{w_2 - w_1}{T} + L_{56}\frac{w_3 - w_1}{T}, \tag{18}$$

while for the tensor

$$\Pi_{ij} = -\frac{L}{T}(\nabla w)_{ij}.$$

We see from (16) that the energy (heat) transfer proceeds both by conduction and as a result of diffusion (the Dufaud effect) and as a result of the action of the "fictitious" forces, as per Zhukovskii. It follows from (17) that the flow of mass of the component is governed not only by the diffusion effect, but also by the conduction of heat (the Soret effects), as well as by the "fictitious" forces.

What remains now is the substitution into (2)-(4), (6)-(8), and (9) and (10) the values of the thermodynamic flows and to derive a closed system of differential equations for the thermodynamic system.

To these equations we have to add the equations of state for the component, as well as the equation for the transfer.

We note that if in the approximate practical calculations we neglect the terms L_{ij} ($i \neq j$) and the diffusion of pressure, Eqs. (16)-(18) yield

$$\begin{aligned} j_q &= -\lambda \operatorname{grad} T; & R_2 &= \frac{m_2^2\rho_2\nu_2}{k_2}(w_2 - w_1); \\ j_1 &= -\rho_1 D_1 \nabla\rho_{10}; & R_3 &= \frac{m_3^2\rho_3\nu_3}{k_3}(w_3 - w_1); \\ j_2 &= -\rho_2 D_2 \nabla\rho_{20}; & \Pi_{ij} &= -\zeta(\nabla w)_{ij}. \\ j_3 &= -\rho_3 D_3 \nabla\rho_{30}; & & \end{aligned} \tag{19}$$

On the basis of (19) Eqs. (2)-(4) assume the form

$$\begin{aligned} &\frac{\partial}{\partial t}(m_2\rho_2\theta_2) + \operatorname{div}(m_2\rho_2\theta_2 w) - \nabla(\rho_2 D_2 \nabla\rho_{20}) - I_{23} = 0; \\ &\frac{\partial}{\partial t}(m_3\rho_3\theta_3) + \operatorname{div}(m_3\rho_3\theta_3 w) - \nabla(\rho_3 D_3 \nabla\rho_{30}) + I_{32} = 0; \\ &\frac{\partial}{\partial t}(m_1\rho_1) + \operatorname{div}(m_1\rho_1 w) - \nabla(\rho_1 D_1 \nabla\rho_{10}) = 0. \end{aligned} \tag{20}$$

The equations of motion (6)-(8) assume the form

$$\rho_2\theta_2\frac{dw_2}{dt} = -\operatorname{grad} P_2 + I_{23}\cdot(w_2 - w_3) - \frac{m_2\rho_2\nu_2}{k_2}(w_2 - w_1) + \rho_2 F_2$$

(analogously for the third component),

$$m_1\rho_1\frac{dw_1}{dt} - \operatorname{div}(m_1\Pi) - (P_2\cdot\operatorname{grad} m_2 + P_3\cdot\operatorname{grad} m_3) = \frac{m_2^2\rho_2\nu_2}{k_2}(w_2 - w_1) + \frac{m_3^2\rho_3\nu_3}{k_3}(w_3 - w_1) + \rho_1 F_1. \tag{21}$$

The equation for the balance for total energy, which follows easily from (9) and (10), has the form

$$\frac{\partial}{\partial t}\left[m_1\rho_1\left(\frac{1}{2}w_1^2+u_1+\Psi_1\right)\right]+\operatorname{div}\left[m_1\rho_1\left(\frac{1}{2}w_1^2+u_1+\Psi_1\right)w_1\right]-\operatorname{div}(m_1\Pi w_1)-\lambda_1\Delta T-\delta A_1=0;$$

$$\frac{\partial}{\partial t}\left[m_2\rho_2\theta_2\left(\frac{1}{2}w_2^2+u_2+\Psi_2\right)\right]+\operatorname{div}\left[m_2\rho_2\theta_2\left(\frac{1}{2}w_2^2+u_2+\Psi_2\right)w_2\right]+\operatorname{div}(m_2P_2w_2)+\operatorname{div}(m_2\theta_2\Psi_2j_2)-\lambda_2\Delta T+\delta A_2=0.$$

The equations for the balance of the intrinsic energy for the components of the system are very easily derivable from (22) and from the equation for the balance of kinetic and potential energy density.

The resulting relationships should be expanded with the equations of state:

$$u_2=f_2(P_2,\ T);\quad P_2=F_2(\rho_2,\ T);$$
$$u_3=f_3(P_3,\ T);\quad P_3=F_3(\rho_3,\ T);$$

with the equation $\sum_{i=1}^{3}\rho_{i0}=1$ and with the formula for the transfer of matter.

As regards the dissipation work, this is easily calculated from the expressions for the energy, derived through averaging over the volume, i.e.,

$$\delta A_2=R_2w_1+P_2\,\frac{\partial(m_2\theta)}{\partial t};$$

$$\delta A_3=R_3w_1+P_3\,\frac{\partial(m_3\theta_3)}{\partial t}.$$

For the nondeformable solid component each of these expressions vanishes.

We have thus derived a closed system of differential interrelated equations for the transfer of mass and energy in the case of a process of nonsteady filtration in crack-porous layers.

NOTATION

$\rho_{i0}=m_i\theta_i\rho_i/\rho$	is the mass concentration of the component ($\theta_1=1$) ($i=1,2,3$);
m_2 and m_3	are the porosities of the blocks and of the crack;
$m_1=1-m_2-m_3$;	
θ_2, θ_3	are the saturation of the porous block and of the crack space with liquid;
ρ_i ($i=1,2,3$)	is the density of the component;
$\rho=\sum_{i=1}^{3}\rho_i$;	
$j_i=m_i\rho_i\theta_i(w_i-w)$	denotes the vector for the diffusion flow of the component i ($i=1,2,3$);
w_i	is the velocity vector for the i-th component;
$w=\sum_{i=1}^{3}m_i\rho_i\theta_iw_i/\sum_{i=1}^{3}m_i\rho_i\theta_i$	is the velocity vector for the center of mass of the system;
I_{23}	is the source for the mass of the second component;
I_{32}	is the source for the mass of the third component;
t	is the time;
P_2 and P_3	denote the pressures of the liquid contained within the porous block and in the crack;
F_i ($i=1,2,3$)	is the body-force vector;
R_i ($i=1,2,3$)	is the Zhukovskii "fictitious" force vector;
Π	is the stress tensor in the solid component of the system;
Ψ_i ($i=1,2,3$)	is the specific potential energy of the component;
j_q^i ($i=1,2,3$)	is the heat-flux vector;
δA_i ($i=1,2,3$)	is the work per unit time of the forces at the internal boundaries of separation between the liquid and solid components;
$\delta A_1=\delta A_2+\delta A_3$;	
q_i ($i=1,2,3$)	is the intensity of the transfer of heat as a consequence of nonidentical component temperatures;
$j_{s,a}$	is the entropy flux vector;
s	is the specific entropy;

I_S	is the intensity of the entropy source;
T_i $(i = 1, 2, 3)$	is the component temperature;
μ_i $(i = 1, 2, 3)$	is the specific Hibbs function of the i-th component;
L_{ij}	denote the phenomenological coefficients;
λ	is the coefficient of thermal conductivity;
k_2 and k_3	denote the permeability of the blocks and cracks respectively;
ν_2 and ν_3	denote, respectively, the kinematic viscosity of the liquid in the blocks and in the cracks;
D_i $(i = 1, 2, 3)$	are the diffusion coefficients;
u_i $(i = 1, 2, 3)$	is the specific intrinsic energy of the component;
∇	is the Hamiltonian;
Δ	is the Laplace operator.

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